Chapter 1

Detection of Unexploded Ordnance: An Introduction

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Abstract In this chapter, we outline the statistical procedures that can be employed for the detection of unexploded ordnance (UXO). Phenomenological modeling is first developed to relate the collected data to a sensor's feature parameters, which in turn allow for physics-based signal processing. Starting with the Bayesian framework, we introduce minimax and robust detection that do not require prior probabilities and distributional information on the measurement uncertainty, respectively. Nonparametric tests that perform well for broad classes of distributions are also presented. Finally, the generalized likelihood ratio test is described as a joint estimation-detection method which first estimates the feature parameters and then tests for the presence-absence of the UXO.

Key words: Detection, Gaussian distribution, likelihood ratio test, modeling, minimax, Neyman-Pearson, nonparametric test, UXO

1.1 Introduction

Unexploded ordnance (UXO) refers to explosive devices that are buried below ground. Magnetometers, electromagnetic induction (EMI) and radar are typically used for sensing UXO. Excavation operations are risky and costly, and therefore false alarms should not exceed some acceptable level.

Detection of UXO involves several stages. Data are collected by sensors, preferably several distinct ones. Model parameters are extracted and refined in which information from one device may help constrain the parameter space of another sensor to minimize uncertainties. One example is the location estimate supplied by a magnetometer serving as a constraint when analyzing the EMI data. The next step is the detection of UXO; that is, distinguishing between UXO and non-UXO objects based on statistical tests performed on the measured parameter values. This may be followed by classification of the UXO type.

In this chapter, we concentrate on various detection techniques that primarily differ in the modeling assumptions. We characterize the problem in the form of two hypotheses described as

$$H_1\colon \text{ UXO present,}$$

$$H_0\colon \text{ UXO absent, or non} - \text{UXO present.}$$

We will assume that the measurement uncertainties are represented by the multivariate Gaussian distribution. We adopt this distribution because of its maximum entropy property and the mathematical convenience it brings along, rather than being justified by empirical observation. However, we will also show methodologies that accommodate variations in the distributional form.

The simple, approximate magnetic-dipole model uses the magnetometer field measurements to determine the UXO depth below ground and the magnetic-dipole orientation [1,9]. The EMI response can be modeled by generalizing the magnetometer model through a tensor that ties the excitation magnetic field and the magnetic dipole moment. Assigning unique magnetic dipoles to distinct components of the same UXO and providing more information, the EMI models work with more parameters such as the ordnance's center location constrained by the magnetometer data, UXO orientation (characterized by a unitary transformation matrix on the magnetization tensor), magnetization induced by ferrous elements, and the EMI resonant frequencies [15]. A multisensor towed array system of magnetometers and EMI sensors is shown to perform with a detection probability greater than 0.95 in [9]. The detection of deeply buried UXO by means of a magnetometer equipped with cone penetrometer technology is investigated in [14].

Ground-penetrating radar (GPR) in conjunction with synthetic aperture radar (SAR) processing can be used from airborne [3] or ground platforms

for UXO location identification. In [4], a GPR in the 50-500 MHz range is deployed along with magnetometers. The SAR processing produces three-dimensional images of possible UXO locations. Further data analysis is needed to decide on the actual UXO presence [2]. For the application of a directional borehole radar, see [11].

1.2 Detection

Let $\mathbf{x}$ denote the feature vector whose elements are the magnetometer or EMI model parameters. The signal measurements received from the sensor is the n-dimensional vector $\mathbf{v}$, which is a function of $\mathbf{x}$. Extraction of $\mathbf{x}$ from $\mathbf{v}$ based on a phenomenological model is known as inversion. As an example, consider the EMI dipole model [15], where $\mathbf{H}'$ denotes the excitation magnetic field and $\mathbf{M}$ is the magnetization tensor. The magnetic dipole moment is $\mathbf{m} = \mathbf{M} \cdot \mathbf{H}'$. Assuming that UXO is aligned along the z-axis,

$$\mathbf{M}(\omega) = \mathbf{z}^T\mathbf{z}\left(m_z(0) + \sum_k \frac{\omega m_{zk}}{\omega - j\omega_{zk}}\right) + (\mathbf{x}^T\mathbf{x} + \mathbf{y}^T\mathbf{y})\left(m_p(0) + \sum_k \frac{\omega m_{pk}}{\omega - j\omega_{pk}}\right) \tag{1.1}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are orthonormal vectors and $m_z(0), m_p(0)$ stand for the magnetization induced by ferrous objects. Keeping only the first terms in each summation in (1.1) is sufficient to have a physics-based signal model [1], where ω_{z1} and ω_{p1} can be used as features in the detection set-up because the imaginary resonant frequencies are functions of the UXO material properties and size.

1.2.1 Bayesian Framework

Let $\pi_i, i = 0, 1$, be the *a priori* probability of $H_i, i = 0, 1$, with $\pi_0 + \pi_1 = 1$. The probability π_1 represents the prior knowledge, expectation or guess regarding the likelihood of encountering an actual UXO. Thus, the higher π_1, the greater the chance of running into an UXO at the area of exploration.

Suppose that we can devise a cost coefficient $c_{ij}, i, j = 0, 1$ which represents the cost of deciding on H_i when H_j is true. Assuming that $c_{11} < c_{01}$, the expected Bayesian risk associated with the choice between the presence and absence of a UXO is minimized by the decision rule δ_B:

$$\delta_B = \begin{cases} 1 \text{ if } L(\mathbf{v}) \geq t, \\ 0 \text{ if } L(\mathbf{v}) < t \end{cases}$$

where the likelihood ratio (LR) and the threshold are respectively defined as

$$L(\mathbf{v}) = \frac{f_1(\mathbf{v})}{f_0(\mathbf{v})},$$

$$t = \frac{(c_{10} - c_{00})\pi_0}{(c_{01} - c_{11})\pi_1}$$

with $f_i(\mathbf{v}) = f(\mathbf{v}|H_i), i = 0, 1$.

Determining the value of π_1 with good accuracy is critical for the detection performance when the Bayesian framework in (1.2.1) is employed. If the estimated π_1 value is overshot, the test will yield more false alarms than necessary. Similarly, an unrealistically low π_1 may result in excessive number of undetected UXOs with possibly catastrophic consequences. Therefore, it is desirable to work with detectors that do not require the priors, or that are insensitive to deviations from true values.

1.2.2 Minimax Solution

Suppose that the prior probabilities (π_0, π_1) are unknown and they cannot be estimated with sufficient precision. A conservative approach to detector design would be to ensure good performance under the "least favorable" conditions, which is characterized by the priors that maximize the Bayesian cost. Such a worst-case design guarantees a minimum performance level in the event of parametric uncertainty: for any other (π_0, π_1) pair, the detector will do even better.

Let $R(\delta|H_i)$ denote the Bayesian risk associated with the decision rule δ given that hypothesis $H_i, i = 0, 1$, is true. For the unknown prior π_0, the expected risk is

$$R(\delta, \pi_0) = R(\delta|H_0)\pi_0 + R(\delta|H_1)(1 - \pi_0).$$

In accordance with the Bayesian paradigm, the goal now is to find a decision rule-least favorable prior pair (δ_M, π_{0M}) which solves the minimax problem:

$$(\delta_M, \pi_{0M}) = \arg\min_{\delta} \max_{\pi_{0M} \in (0,1)} R(\delta, \pi_0). \tag{1.2}$$

The formulation in (1.2) can be viewed as a competitive game between the engineer and nature. While the engineer attempts to minimize the cost by designing the best detector, nature tries to maximize the penalty involved by selecting the least favorable prior. From the engineer's perspective, nature

wants to maximize the minimum cost induced by his/her δ decision. In contrast, the engineer's objective is to minimize the maximum cost that occurs from the (π_0, π_1) selection. The pair (δ_M, π_{0M}) exhibits a so-called saddle point behavior described by

$$R(\delta_M, \pi_0) \leq R(\delta_M, \pi_{0M}) \leq R(\delta, \pi_{0M})$$

for all δ and π_0.

The saddle point property stipulates that the following condition is satisfied for any $R(\delta, \pi_0)$.

$$R(\delta_M, \pi_{0M}) = \max_{\pi_0 \in (0,1)} \min_{\delta} R(\delta, \pi_0) = \min_{\delta} \max_{\pi_0 \in (0,1)} R(\delta, \pi_0). \tag{1.3}$$

The interpretation of (1.3) is simple and useful: If (δ_M, π_{0M}) is a saddle point, then it solves the minimax problem in (1.2), and vice versa. Moreover, the minimax solution is the same as the maximin solution, and one can opt for one or the other depending on their relative ease and complexity.

1.2.3 Neyman-Pearson Framework

In addition to the lack of reliable information about the prior probabilities, it may not be possible to formulate meaningful cost coefficients as required by the Bayesian set-up. The two error types may have asymmetrical penalties involved. Specifically, let

$$e_{\mathrm{I}}(\delta) = P\{H_1 | H_0 \,\text{is true}\}$$

which is known as the false alarm probability, or type I error in statistics. Similarly, the miss probability, or type II error is defined as

$$e_{\mathrm{II}}(\delta) = P\{H_0 | H_1 \,\text{is true}\}.$$

It is clear that a false alarm event merely triggers a costly UXO removal operation whereas a miss leaves the UXO undetected. It is impossible to minimize e_{I} and e_{II} simultaneously. Recognizing that minimizing misses is far more important than avoiding false alarms leads to the following constraint optimization problem:

$$\text{Minimize } e_{\mathrm{II}}(\delta) \,\text{subject to}\, e_{\mathrm{I}}(\delta) \leq \alpha. \tag{1.4}$$

Note that without the bound on the false alarm probability, one could achieve $e_{\mathrm{II}}(\delta) = 0$ by simply having $\delta = 1$ at all times. Unfortunately, this is an infeasible solution because it requires infinite time and resource budgets.

The solution of (1.4), which follows the construction of the appropriate Lagrangian and the application of Kuhn-Tucker conditions, is stated in the Neyman-Pearson Lemma.

$$\delta_{NP} = \begin{cases} 1 & \text{if } L(\mathbf{v}) > \lambda(\alpha), \\ r(\alpha) & \text{if } L(\mathbf{v}) = \lambda(\alpha), \\ 0 & \text{if } L(\mathbf{v}) < \lambda(\alpha), \end{cases}$$

where the threshold $\lambda(\alpha)$ and the randomization constant $r(\alpha) \in [0,1]$ are such that

$$\int_{\mathcal{V}_1} f_0(\mathbf{v})d\mathbf{v} + r(\alpha)\int_{\mathcal{V}_2} f_0(\mathbf{v})d\mathbf{v} = \lambda(\alpha),$$

and

$$\mathcal{V}_1 = \left\{ \mathrm{v} \colon \frac{f_1(\mathrm{v})}{f_0(\mathrm{v})} > \lambda(\alpha) \right\},$$

$$\mathcal{V}_2 = \left\{ \mathrm{v} \colon \frac{f_1(\mathrm{v})}{f_0(\mathrm{v})} = \lambda(\alpha) \right\}.$$

Once again, the optimal detector takes the form of a likelihood ratio test but in Neyman-Pearson set-up, the threshold is determined by the false alarm rate α, instead of priors or cost coefficients. If the density functions $f_i(\mathrm{v}), i = 0, 1$, are continuous everywhere, then randomization is not necessary and $r(\alpha)$ can be set to unity.

1.3 Gaussian Uncertainty

Suppose that the measurement uncertainties are represented by the multivariate Gaussian random variable so that

$$f_i(\mathbf{v}) = \frac{1}{(2\pi)^{n/2}|\mathbf{\Sigma}_i|^{1/2}} \exp\left\{ -\frac{1}{2}(\mathbf{v} - \mu_i)^T \mathbf{\Sigma}_i^{-1}(\mathbf{v} - \mu_i) \right\}$$

where $\mathbf{m}_i$ and $\mathbf{\Sigma}_i, i = 0, 1$, are respectively the mean vector and the covariance matrix under $H_i, i = 0, 1$. The natural logarithm of the likelihood ratio takes the form

$$L'(\mathbf{v}) = \log_e L(\mathbf{v}) = (\mathbf{v}-\mathbf{m}_0)^T\mathbf{\Sigma}_0^{-1}(\mathbf{v}-\mathbf{m}_0) - (\mathbf{v}-\mathbf{m}_1)^T\mathbf{\Sigma}_1^{-1}(\mathbf{v}-\mathbf{m}_1). \quad (1.5)$$

The degree of correlation between successive measurements is hard to determine, and as an approximation and to keep the design simple, one can assume that $\mathbf{\Sigma}_i = \mathrm{diag}\{\sigma_{i1}^2, \sigma_{i2}^2, \ldots, \sigma_{in}^2\}, i = 0, 1$. Letting $\mathbf{m}_i = [m_{i1} \ \cdots \ m_{in}]^T$ and $\mathbf{v}_i = [v_{i1} \ \cdots \ v_{in}]^T$, (1.5) becomes

$$L'(\mathbf{v}) = \sum_{k=1}^{n} \frac{(v_{0k} - m_{0k})^2}{\sigma_{0k}^2} - \sum_{k=1}^{n} \frac{(v_{1k} - m_{1k})^2}{\sigma_{1k}^2}$$

The optimal Bayesian test is

$$\delta_{B,G} = \begin{cases} 1 \text{ if } L'(\mathbf{v}) \geq t', \\ 0 \text{ if } L'(\mathbf{v}) < t' \end{cases}$$

where

$$t' = \log_e \frac{(c_{10} - c_{00})\pi_0}{(c_{01} - c_{11})\pi_1}.$$

If there is sufficient evidence that the underlying uncertainty cannot be adequately described by the Gaussian distribution, then it is possible to resort to robust formulations that ensure good performance even if there are deviations from the nominally assumed probability distribution. Let $\mathcal{F}_0$ and $\mathcal{F}_1$ respectively denote two disjoint classes of multivariate probability density functions (PDFs) that represent H_0 and H_1. Following a similar game as in the minimax construction, we then seek to find a pair $(\delta^*, f_1^*(\mathbf{v}))$, where δ^* is an admissible decision rule and $f_1^*(\mathbf{v}) \in \mathcal{F}_1$, such that

$$e_{\text{II}}(\delta, f_1^*) \leq e_{\text{II}}(\delta^*, f_1^*) \leq e_{\text{II}}(\delta^*, f_1), \forall \delta \in \mathcal{D}, \forall f_1 \in \mathcal{F}_1, \tag{1.6}$$

and

$$e_{\text{I}}(\delta^*, f_0) \leq \alpha, \forall f_0 \in \mathcal{F}_0, \tag{1.7}$$

where $\mathcal{D}$ is the class of admissible decision rules and α is a prespecified false alarm rate. If there exists a δ^* that satisfies (1.6) and (1.7), then it is referred to as a robust rule. The pair of density functions, f_0^* and f_1^* that satisfy (1.6) and (1.7) for the rule δ^* are called least favorable in $\mathcal{F}_0 \cup \mathcal{F}_1$. Moreover, δ^* is clearly a Neyman-Pearson test at (f_0^*, f_1^*) and α.

An interesting special case where $\mathcal{F}_0$ and $\mathcal{F}_1$ represent the following classes of stationary and memoryless processes has been extensively studied by Huber [6].

$$\mathcal{F}_0 = \{f(v) = (1 - \varepsilon_0) f_0(v) + \varepsilon_0 h(v),\ v \in \mathcal{R}, h \in \mathcal{H}\}, \tag{1.8}$$

$$\mathcal{F}_1 = \{f(v) = (1 - \varepsilon_1) f_1(v) + \varepsilon_1 h(v),\ v \in \mathcal{R}, h \in \mathcal{H}\} \tag{1.9}$$

where $\mathcal{R}$ is the real line, $\mathcal{H}$ is the class of all density functions, $\varepsilon_0, \varepsilon_1 \in (0, 1)$ and v is some element of the feature vector $\mathbf{v}$ [7]. The robust detector under the classes of distributions defined in (1.8) and (1.9) is the likelihood ratio test designed for the corresponding least favorable f_0^*, f_1^*, which trims data that exceed certain threshold values, thereby eliminating the outliers.

1.4 Nonparametric Detection

The measurement and modeling uncertainty is specified by a relatively restricted family of distributions in (1.8) and (1.9). A detector that performs well for a broader class arises from the nonparametric procedures. Let $\mathcal{F}_{-\theta}$ represent the class of stationary and memoryless discrete-time processes with common mean at $-\theta$. Each member of the class is denoted by the first-order probability density function $f_{-\theta}$. The hypotheses H_1 and H_0 are described by $\mathcal{F}_\theta$ and $\mathcal{F}_{-\theta}$, respectively. A decision rule consists of the triplet $(T(\mathbf{v}), \lambda, r)$ where $T(\mathbf{v})$ is the corresponding test function, α is the preset false alarm rate and r is the randomization constant. The rule or test $(T(\mathbf{v}), \lambda, r)$ is nonparametric in $(\mathcal{F}_\theta, \mathcal{F}_{-\theta})$ if and only if it induces the same false alarm probability for all $f_{-\theta} \in \mathcal{F}_{-\theta}$ [7].

Let $\mathcal{F}$ denote the class of distributions obtained from either $\mathcal{F}_\theta$ or $\mathcal{F}_{-\theta}$ when the mean is set to zero for all members. For some $f \in \mathcal{F}$, suppose that f_θ is the PDF induced by f when its mean is changed from zero to θ. Let $n(\alpha, \beta, T_{f_\theta})$ be the number of data required by a Neyman-Pearson rule to attain the detection probability (also known as the power of the test) β while satisfying the false alarm constraint α when testing f_θ against $f_{-\theta}$. Likewise, let $n(\alpha, \beta, T, f_\theta, f_{-\theta})$ be the sample size needed by the nonparametric test of f_θ versus $f_{-\theta}$ to achieve the power β with false alarm α. The efficacy, EFF, and the asymptotic relative efficiency, ARE, are defined as

$$\mathrm{EFF} = \lim_{n\to\infty} \frac{\left(\frac{\partial}{\partial\theta} E[T(\mathbf{v})|f_\theta]\big|_{\theta=0}\right)^2}{n \cdot \mathrm{var}(T(\mathbf{v})|f_\theta)},$$

and

$$\mathrm{ARE} = \lim_{\theta\to 0} \frac{n(\alpha, \beta, T_{f_\theta})}{n(\alpha, \beta, T, f_\theta, f_{-\theta})}.$$

Efficacy indicates the asymptotic discrimination ability of the test when the hypotheses are close to each other. Asymptotic relative efficiency measures the additional sample size needed by the nonparametric test to yield the same power as the optimal Neyman-Pearson rule when the two hypotheses are asymptotically close to each other.

1.4.1 Sign Test

The sign test was originally introduced as an ad hoc formalization but it also evolves as a limiting case of the robust test for the classes in (1.8) and (1.9). The associated test function is

$$T(\mathbf{v}) = \frac{1}{n}\sum_{i=1}^{n} \operatorname{sgn} v_i$$

where

$$\operatorname{sgn} v_i = \begin{cases} 1 \text{ if } v_i > 0, \\ 0 \text{ if } v_i \leq 0. \end{cases}$$

The sign test is nonparametric in $(\mathcal{F}_\theta, \mathcal{F}_{-\theta})$ for any n, and for f_θ and $f_{-\theta}$ generated by Gaussian PDF with variance σ^2, its efficacy and asymptotic relative efficiency are EFF $= 8/\pi\sigma^4$ and ARE $= 2/\pi$ [7]. Thus, the sign test requires about 57% more samples to reach the same performance level as the Gaussian-optimal Neyman-Pearson rule (as $\theta \to 0$), but the latter experiences performance degradation when the Gaussian distribution is not actually a valid uncertainty model.

1.4.2 Optimal Rank Test

The rank tests first order the measurements $\{v_1, \ldots, v_n\}$ from smallest to the largest and then take the signs of the ordered data. The new vector $\mathbf{z} = [z_1 \ \cdots \ z_n]^T$ of the signs, where

$$z_i = \begin{cases} 1 \text{ if the } i\text{th ranked datum in } \mathbf{v} \text{ has nonnegative sign}, \\ 0 \text{ if the } i\text{th ranked datum in } \mathbf{v} \text{ has negative sign}, \end{cases}$$

is called the rank vector.

Given $\theta > 0$ and some $f \in \mathcal{F}$, the optimal-at-f rank test is

$$\delta_O = \begin{cases} 1 \text{ if } \frac{K_{f_\theta}(\mathbf{z})}{K_{f_{-\theta}}(\mathbf{z})} > \lambda, \\ r \text{ if } \frac{K_{f_\theta}(\mathbf{z})}{K_{f_{-\theta}}(\mathbf{z})} = \lambda, \\ 0 \text{ if } \frac{K_{f_\theta}(\mathbf{z})}{K_{f_{-\theta}}(\mathbf{z})} < \lambda \end{cases}$$

where

$$K_{f_\theta}(\mathbf{z}) = n! \int \cdots \int \prod_{i=1}^{n} f(v_i - \theta z_i) d\mathbf{v},$$

$$K_{f_{-\theta}}(\mathbf{z}) = K_{f_\theta}(-\mathbf{z}),$$

and λ and r satisfy

$$P\left\{ \frac{K_{f_\theta}(\mathbf{z})}{K_{f_{-\theta}}(\mathbf{z})} > \lambda | f_{-\theta} \right\} + r \cdot P\left\{ \frac{K_{f_\theta}(\mathbf{z})}{K_{f_{-\theta}}(\mathbf{z})} = \lambda | f_{-\theta} \right\} = \alpha.$$

For the optimal-at-f rank test, ARE = 1, and the efficacy at f is the Fisher information, i.e.,

$$\text{EFF} = \int_{\mathcal{R}} \frac{[f'(x)]^2}{f(x)} dx$$

so long as $f \in \mathcal{F}$ possesses a Taylor series expansion [7].

1.4.3 Wilcoxon Rank Test

The Wilcoxon rank test [12] is as follows.

$$\delta_W = \begin{cases} 1 \text{ if } \sum_{i=1}^n iz_i > \lambda, \\ r \text{ if } \sum_{i=1}^n iz_i = \lambda, \\ 0 \text{ if } \sum_{i=1}^n iz_i < \lambda, \end{cases}$$

with λ and r such that

$$\sup_{f_{-\theta} \in \mathcal{F}_{-\theta}} \left(P\left\{ \sum_{i=1}^n iz_i > \lambda | f_{-\theta} \right\} + r \cdot P\left\{ \sum_{i=1}^n iz_i = \lambda | f_{-\theta} \right\} \right) = \alpha.$$

While the optimal rank test is the Neyman-Pearson test for the rank vector $\mathbf{z}$ extracted from a particular $f \in \mathcal{F}$, the Wilcoxon test is designed for the entire classes $(\mathcal{F}_\theta, \mathcal{F}_{-\theta})$. For Gaussian PDF with variance σ^2, the Wilcoxon rank test has ARE $= 3/\pi < 1$ and EFF $= 6/\pi\sigma^2$ [7].

The tests described in this section are nonparametric in both $(\mathcal{F}_\theta, \mathcal{F}_{-\theta})$ and $(\mathcal{F}_{2\theta}, \mathcal{F})$, which ensures broader applicability.

1.5 Generalized Likelihood Ratio Test

So far we have assumed that model parameters were known. In practice, these have to be estimated beforehand or simultaneously with the detection procedure. In the latter case, the generalized likelihood ratio test (GLRT) offers a joint estimation and detection methodology to solve the composite hypotheses that are represented by the corresponding density functions as below.

$$H_1 \colon f_1(\mathbf{v}|\mathbf{x}), \mathbf{x} \in \mathcal{X}_1,$$
$$H_0 \colon f_0(\mathbf{v}|\mathbf{x}), \mathbf{x} \in \mathcal{X}_0.$$

Suppose that $\hat{\mathbf{x}}_i$ is the maximum likelihood estimate of $\mathbf{x}$ under $H_i, i = 0, 1$, i.e., $\hat{\mathbf{x}}_i = \arg\max_{\mathbf{x}\in\mathcal{X}_i}$. Then, the generalized likelihood ratio (GLR) is

$$L_{\mathrm{GLR}}(\mathbf{v}) = \frac{f_1(\mathbf{v}|\hat{\mathbf{x}}_1)}{f_0(\mathbf{v}|\hat{\mathbf{x}}_0)}.$$

If the null hypothesis H_0 is the absence of UXO, then $\mathbf{x} = \mathbf{x}_0$ with no need for estimation. The GLRT is

$$\delta_{\mathrm{GLRT}} = \begin{cases} 1 \text{ if } L_{\mathrm{GLR}}(\mathbf{v}) \geq \lambda(\alpha), \\ 0 \text{ if } L_{\mathrm{GLR}}(\mathbf{v}) < \lambda(\alpha) \end{cases}$$

where $\lambda(\alpha)$ is such that

$$\max_{\mathbf{x}\in\mathcal{X}_0} e_{\mathrm{I}}(\delta_{\mathrm{GLRT}}, \mathbf{x}) = \alpha$$

with the type I error now defined as

$$e_{\mathrm{I}}(\delta_{\mathrm{GLRT}}, \mathbf{x}) = P\{L_{\mathrm{GLR}}(\mathbf{v}) > \lambda(\alpha)|\mathbf{x}, H_0 \text{ is true}\} = \alpha.$$

Model inversion to estimate $\mathbf{x}$ from the measurements is sensitive to errors in sensor positions, and Bayesian methods employing the GLRT that improve the detection performance are proposed in [10].

1.6 Conclusion

We have provided a brief summary of the statistical tools that are available for the detection of UXO. The application of the tests require accurate models that relate various UXO parameters to observed data, as well as a good distributional description for the uncertainties in measurements and modeling. Performance can be assessed by plotting e_{II} versus e_{I}, the so-called receiver operating characteristic (ROC). The ROC curve gives the detection probability for a given false alarm rate. To combine detection and ordnance classification, it is possible to set up a multiple hypothesis testing problem where each hypothesis $H_i, i \neq 0$, corresponds to a possible UXO type with the associated feature vector, and H_0 still represents the absence of UXO. The need for accurate representation of measurement uncertainties can be alleviated by using model-free approaches such as the support vector machine which relies on preprocessing with training data [15]. For multiple closely spaced UXO parts, blind source separation methods such as independent component analysis can precede the UXO detector [5]. Deploying multiple sensors with distinct distances from the buried UXO can deliver significant classification performance improvement as demonstrated in [13]. The reader is referred to

the bibliography and the references therein for further exploration into UXO detection.

References

1. L. Carin, H. Yu, Y. Dalichaouch, A. R. Perry, P. V. Czipott and C. E. Baum, "On the wideband EMI response of a rotationally symmetric permeable and conducting target," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 39, No. 6, pp. 1206-1213, June 2001.
2. C.-C. Chen and L. Peters, "Buried unexploded ordnance identification via complex natural resonances," *IEEE Transactions on Antennas and Propagation*, Vol. 45, No. 11, pp. 1645-1654, November 1997.
3. A. Fijany, J. B. Collier and A. Citak, "Recent advances in unexploded ordnance (UXO) detection using airborne ground penetrating SAR," *Proceedings of the Aerospace Conference*, 1999, pp. 429-441.
4. J. I. Halman, K. A. Shubert and G. T. Ruck, "SAR processing of ground-penetrating data for buried UXO detection: Results from a surface-based system," *IEEE Transactions on Antennas and Propagation*, Vol. 46, No. 7, pp. 1023-1027, July 1998.
5. W. Hu, S. L. Tantum and L. M. Collins, "EMI-based classification of multiple closely spaced subsurface objects via independent component analysis," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 42, No. 11, pp. 2544-2554, September 2004.
6. P. J. Huber, *Robust Statistics*, New York, NY: Wiley, 1981.
7. D. Kazakos and P. Papantoni-Kazakos, *Detection and Estimation*, New York, NY: Computer Science Press, 1990.
8. C. V. Nelson, C. C. Cooperman, W. Schneider, D. S. Wenstrand and D. G. Smith, "Wide bandwidth time-domain electromagnetic sensor for metal target classification," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 39, No. 6, pp. 1129-1138, June 2001.
9. H. H. Nelson and J. R. MacDonald, "Multisensor towed array detection system for UXO detection," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 39, No. 6, pp. 1139-1145, June 2001.
10. S. L. Tantum, Y. Yu and L. M. Collins, "Bayesian mitigation of sensor position errors to improve unexploded ordnance detection," *IEEE Geoscience and Remote Sensing*

Letters, Vol. 5, No. 1, pp. 103-107, January 2008.

11. R. van Waard, S. van der Baan and K. W. A. van Dongen, "Experimental data of a directional borehole radar system for UXO detection," *Proceedings of the Tenth International Conference on Ground Penetrating Radar*, Delft, The Netherlands, June 2004, pp. 225-228.
12. F. Wilcoxon, "Individual comparisons by ranking methods," *Biometrics*, Vol. 1, 1945, pp. 80-83.
13. D. Williams, C. Wang, X. Liao and L. Carin, "Classification of unexploded ordnance using incomplete multisensor multiresolution data," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 45, No. 7, pp. 2364-2373, July 2007.
14. Q. Zhang, W. Al-Muaimy and Y. Huang, "Detection of deeply buried UXO using CPT magnetometers," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 45, No. 2, pp. 410-417, February 2007.
15. Y. Zhang, L. Collins, H. Yu, C. E. Baum and L. Carin, "Sensing of unexploded ordnance with magnetometer and induction data: theory and signal processing," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 41, No. 5, pp. 1005-1015, May 2003.