Multifractal analysis of images: New connexions between analysis and geometry

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Natural images can be modelled as patchworks of homogeneous textures with rough contours. The following stages play a key role in their analysis:

1. Separation of each component
2. Characterization of the corresponding textures
3. Determination of the geometric properties of their contours

Multifractal analysis proposes to classify functions by using as relevant parameters the dimensions of their sets of singularities. This framework can be used as a classification tool in the last two steps enumerated above. Several variants of multifractal analysis were introduced, depending on the notion of singularity which is used. We will describe the variants based on Hölder and L^p regularity, and we will apply these notions to the study of functions of bounded variation (indeed the BV setting is a standard functional assumption for modelling images, which is currently used in the first step for instance). We will also develop a multifractal analysis adapted to contours, where the regularity exponent associated with points of the boundary is based on an accessibility condition. This analysis can be used in the classification of domains with fractal boundaries.