PONS, REED-MULLER CODES, AND GROUP ALGEBRAS

Myoung An, Jim Byrnes, William Moran, B. Saffari, Harold S. Shapiro, and Richard Tolimieri
Prometheus Inc.
21 Arnold Avenue
Newport, RI 02840
jim@prometheus-inc.com

Abstract In this work we develop the family of Prometheus orthonormal sets (PONS) in the framework of certain abelian group algebras. Classical PONS, considered in 1991 by J. S. Byrnes, turned out to be a rediscovery of the 1960 construction by G. R. Welti, and of subsequent rediscoveries by other authors as well.

This construction highlights the fundamental role played by group characters in the theory of PONS. In particular, we will relate classical PONS to idempotent systems in group algebras and show that signal expansions over classical PONS correspond to multiplications in the group algebra.

The concept of a splitting sequence is critical to the construction of general PONS. We will characterize and derive closed form expressions for the collection of splitting sequences in terms of group algebra operations and group characters.

The group algebras in this work are taken over direct products of the cyclic group of order 2. PONS leads to idempotent systems and ideal decompositions of these group algebras. The relationship between these special systems and ideal decompositions, and the analytic properties of PONS, is an open research topic. A second open research topic is the extension of this theory to group algebras over cyclic groups of order greater than 2.

Keywords: character basis, companion row, crest factor, FE, Fejer dual, functional equation, generating function, generating polynomial, Golay, group algebra, Hadamard matrix, PONS, QMF, Reed-Muller Codes, Shapiro transform, Shapiro sequence, splitting property, splitting sequence, symmetric PONS, Thue-Morse sequence, Welti codes, Walsh-Hadamard matrices.